

# Indian Maritime University

( A Central University, Govt of India)

May-June 2018 End Semester Examinations

## B. Tech (Marine Engineering)

Semester-I

### Mathematics I (UG11T2102 / T1102)

Date: 05.07.2018

Time: 3 Hrs

Max Marks:100 Marks

Pass Marks:50 Marks

**PART-A**

**Marks: 10 x 3 =30**

**(All Questions are compulsory)**

1. a. Find the  $n^{\text{th}}$  derivative of  $\cos x \cos 2x \cos 3x$

b. Verify Rolle's theorem for  $f(x) = \log\left(\frac{x^2+ab}{x(a+b)}\right)$  in  $(a, b)$

c. If  $u = x \log(xy)$  where  $x^3 + y^3 + 3xy = 1$  find  $\frac{du}{dx}$ .

d. Evaluate

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy dx}{1+x^2+y^2}$$

e. Evaluate

$$\int_0^1 x^3(1-\sqrt{x})^5 dx$$

f. Evaluate

$$\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$$

by changing to Polar co-ordinates.

g. Find the extremals of the functional

$$\int_{x_2}^{x_1} \frac{y^{1^2}}{x^3} dx$$

- h. Find an unit vector normal to the surface  $xy^3z^2 = 4$  at the point  $(-1, -1, 2)$
- i. If  $\bar{A}$  and  $\bar{B}$  are irrotational, prove that  $\bar{A} \times \bar{B}$  is solenoidal.
- j. Find the eigen values of the matrix  $\begin{bmatrix} 4 & 3 \\ 3 & 9 \end{bmatrix}$

**PART-B**

**Marks: 5 x 14 = 70**

**(Answer any 5 of the following 7 questions)**

2. a. If  $y = (x^2 - 1)^n$ ,  
prove that  $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0$
- b. Find the coordinates of the centre of curvature at  $(at^2, 2at)$  on the parabola  $y^2 = 4ax$   
(7 + 7 marks)
3. a. Examine the following functions for extreme values  
$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$$
- b. If  $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + \log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ ,  
find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$   
(7 + 7 marks)
4. a. Change the order of integration and evaluate

$$\int_0^a \int_y^a \frac{x \, dx \, dy}{x^2 + y^2}$$

- b. Express

$$\int_0^1 x^m (1 - x^n)^p dx$$

in terms of gamma function.

(7 + 7 marks)

5. a. Show that

$$\int_0^\pi \frac{\log(1 + \alpha \cos x)}{\cos x} dx = \pi \sin^{-1} \alpha$$

- b. Prove that the shortest distance between 2 points in a plane is a straight line.

(7 + 7 marks)

6. a. Find the values of a and b such that the surface

$ax^2 - byz = (a + 2)x$  and  $4x^2y + z^3 = 4$  cut orthogonally at  $(1, -1, 2)$

- b. If  $u\vec{F} = \vec{V}v$  where u and v are scalar fields and  $F$  is a vector field, show that  $\vec{F} \text{ curl } \vec{F} = 0$ .

(7 + 7 marks)

7. a. Find the orthogonal trajectories of the family of

curves  $x^4 + y^4 - 6x^2y^2 = \text{Constant}$

- b. Evaluate, using Cauchy's integral formula

$$\oint_c \frac{z}{z^2 - 3z + 2} dz$$

where c is  $|z - 2| = \frac{1}{2}$

(7 + 7 marks)

8. a. Using Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \text{ express } A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$$

as a linear polynomial in  $A$ .

b. Given that  $\bar{A} = \begin{bmatrix} 0 & 1 + 2i \\ -1 + 2i & 0 \end{bmatrix}$  show that

$(I - A)(I + A)^{-1}$  is a unitary matrix.

(7 + 7 marks)

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